AS EQUAÇÕES DE LAPLACE E DE POISSON
THE EQUATIONS OF LAPLACE AND POISSON

1) Dois planos condutores **A** e **B** estão paralelos entre si e localizados no referencial cartesiano em **x**=0 e em **x**=0,2m, respectivamente. O potencial de referência é em **x**=0,1m. Sabendo que D entre os planos vale 500nC/m^2 segundo o eixo **x**, calcule o potencial dos planos. Two flat conductors **A** and **B** are parallel to each other and located on the Cartesian reference **x**=0 and **x**=0.2m, respectively. The reference potential is at **x** = 0.1m. Knowing that the value of **D** between the planes is 500nC/m^2 along the **x** axis, find the potential at the plan conductors.

$$\nabla^2 \mathrm{V} = 0 \qquad lap(\mathrm{A}) = \nabla^2 \mathrm{A} = \frac{\partial^2 \mathrm{A}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{A}}{\partial \mathrm{y}^2} + \frac{\partial^2 \mathrm{A}}{\partial \mathrm{z}^2} \qquad \mathrm{D} = \varepsilon \mathrm{E}$$

Resolução Resolution

$$\nabla^2 \mathrm{V} = \frac{\partial^2 \mathrm{V}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{V}}{\partial \mathrm{y}^2} + \frac{\partial^2 \mathrm{V}}{\partial \mathrm{z}^2} \qquad \nabla^2 \mathrm{V} = \frac{\partial^2 \mathrm{V}}{\partial \mathrm{x}^2} + 0 + 0 = 0 \qquad \frac{\partial^2 \mathrm{V}}{\partial \mathrm{x}^2} = 0 \qquad \mathrm{V} = -\mathrm{Ex} + \mathrm{V}_0$$

$$\mathrm{V} = -\frac{D}{\varepsilon}\mathrm{x} + \mathrm{V}_0 \qquad 0 = -\frac{D}{\varepsilon}0.1 + \mathrm{V}_0 \qquad \mathrm{V}_0 = \frac{D}{\varepsilon}0.1$$

$$\mathrm{V} = -\frac{D}{\varepsilon}\mathrm{x} + \frac{D}{\varepsilon}0.1 \qquad \mathrm{V} = \frac{D}{\varepsilon}(0.1 - \mathrm{x})$$

$$\mathrm{V}_1 \cong \frac{500\times10^{-9}}{8.854\times10^{-12}}(0.1-0) \qquad \mathrm{V}_1 \cong 5647 \text{ V}$$

$$\mathrm{V}_2 \cong \frac{500\times10^{-9}}{8.854\times10^{-12}}(0.1-0.2) \qquad \mathrm{V}_2 \cong -5647 \text{ V}$$

Resposta Answer

$\mathrm{V}_1 \cong 5647$ V

$\mathrm{V}_2 \cong -5647$ V

2) Dois discos paralelos distam 10mm entre si e estão a uma diferença de potencial entre eles de 400V. A permissividade relativa do dieléctrico é 4 e a área de cada um dos discos é de 4cm2. Two equal parallel discs with area of is 4cm^2 each one are distant from each other of 10mm. The electrical potential difference between them is of 400V. The relative permittivity of the dielectric is 4.

$$\nabla^2 \mathrm{V} = 0 \qquad lap(\mathrm{A}) = \nabla^2 \mathrm{A} = \frac{\partial^2 \mathrm{A}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{A}}{\partial \mathrm{y}^2} + \frac{\partial^2 \mathrm{A}}{\partial \mathrm{z}^2} \qquad \mathrm{D} = \varepsilon \mathrm{E}$$

a) Determine a densidade de cargas nos discos. Find the charge density on the disks.

Resolução Resolution

$$\nabla^2 \mathrm{V} = \frac{\partial^2 \mathrm{V}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{V}}{\partial \mathrm{y}^2} + \frac{\partial^2 \mathrm{V}}{\partial \mathrm{z}^2} \qquad \nabla^2 \mathrm{V} = \frac{\partial^2 \mathrm{V}}{\partial \mathrm{x}^2} + 0 + 0 = 0$$

$$\frac{\partial^2 \mathrm{V}}{\partial \mathrm{x}^2} = 0 \qquad \mathrm{V} = -\mathrm{Ex} + \mathrm{V}_0$$

$$\mathrm{V}_1 = -\mathrm{Ex} + \mathrm{V}_0 \qquad \mathrm{V}_1 = -\mathrm{Ex} + \mathrm{V}_0$$
$$\mathrm{V}_2 = -\mathrm{E}(\mathrm{x} + 10\times10^{-3}) + \mathrm{V}_0 \qquad \mathrm{V}_1 + 400 = -\mathrm{E}(\mathrm{x} + 10\times10^{-3}) + \mathrm{V}_0$$

$$400 = -\mathrm{E}\times10\times10^{-3} \qquad \mathrm{E} = -40\times10^3 \ \mathrm{V/m}$$

$$D = \varepsilon \mathrm{E} \cong -40\times10^3 \times 4 \times 8.854\times10^{-12} \qquad D = \varepsilon \mathrm{E} \cong -1417 \ \mathrm{nC/m^2}$$

$$\vec{\mathrm{E}} = \frac{\rho_s}{2\varepsilon}\vec{a}_R \qquad \vec{\mathrm{E}}\varepsilon = \frac{\rho_s}{2}\vec{a}_R \text{ (um dos discos one of the discs)}$$

$\mathrm{E}\varepsilon = \rho_s$ (para os dois discos for the tow discs)

$\rho_1 = +1417 \ \mathrm{nC/m^2}$

$\rho_2 = -1417 \ \mathrm{nC/m^2}$

Resposta Answer

$\rho_1 = +1417 \ \mathrm{nC/m^2}$ para um dos discos for one of the discs

$\rho_2 = -1417 \ \mathrm{nC/m^2}$ para o outro disco for the other disc

b) Determine o módulo da carga eléctrica em cada um dos discos. Find the magnitude of the electric charge on each disc.

$$\oint \vec{D} \cdot d\vec{S} = Q_{int}$$

Resolução Resolution

$$D \times S = Q_{int} \qquad Q_{int} = 1417 \times 10^{-9} \times 4 \times 10^{-4} \qquad Q_{int} = 566.8 \times 10^{-12}\,C$$

Resposta Answer

$$Q_{int} = 566.8\,pC$$

c) Determine a capacidade formada pelos discos. Find the capacity formed by the discs.

$$C = \frac{Q}{V} \qquad C = \varepsilon \frac{A}{d}$$

Resolução Resolution

$$C = \frac{566.8\,pC}{400\,V} \qquad C = 1.417\,pF$$

ou or

$$C = 4 \times 8.854 \times 10^{-12} \frac{4 \times 10^{-4}}{10 \times 10^{-3}} \qquad C = 1.417\,pF$$

Resposta Answer

$$C = 1.417\,pF$$